Coherent bremsstrahlung at the B factories SLAC PEP-II and KEK Tristan-B

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Coherent bremsstrahlung (CBS) is a specific type of radiation at colliders with short bunches. In the present paper we calculate the main characteristics of CBS for the planned projects of the B factories PEP-II and Tristan-B. At these colliders $dN_\gamma \sim (10^7 - 10^8)\; dE_\gamma/E_\gamma$ photons of CBS will be emitted for a single collision of the beams in the energy range $E_\gamma \lesssim 10$ keV. It seems that CBS can be a potential tool for optimizing collisions and for measuring beam parameters. Indeed, the bunch length σ_z can be found from the CBS spectrum because the critical energy $E_c \propto 1/\sigma_z$; the horizontal transverse bunch size σ_x is related to $dN_\gamma \propto 1/\sigma_x^2$. Besides, CBS may be very useful for a fast control over an impact parameter R between the colliding bunch axes because a dependence of dN_γ on R has a very specific behavior.

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I. THREE TYPES OF RADIATION AT COLLIDERS

Let us speak, for definiteness, about emission by electrons moving through a positron bunch. [We denote the following: N_e and N_p are the numbers of particles in the electron and positron bunches, σ_z, σ_x, and σ_y are the longitudinal, horizontal, and vertical transverse sizes of the positron bunch, $\gamma_e = E_e/m_ec^2$ is the electron Lorentz factor, $E_c = 4\gamma_e^2\hbar c/\sigma_z$ is the characteristic (critical) energy for the coherent bremsstrahlung (CBS) photons, and $r_e = e^2/m_ec^2$.] If the photon energy is large enough, one deals with the ordinary (incoherent) *bremsstrahlung.*

If the photon energy becomes small enough, the radiation is determined by the interaction of the electron with the collective electromagnetic field of the positron bunch. It is known (see, e.g., Sec. 77 in Ref. [1]) that the properties of this coherent radiation are quite different depending on whether the electron deflection angle θ_d is large enough or rather small as compared with the typical emission angle $\sim 1/\gamma_e$. It is not difficult to estimate that

$$\theta_d \sim \eta/\gamma_e,\; \eta = r_eN_p/\sigma_x. \quad (1)$$

We call a positron bunch *long* if $\eta \gg 1$. The radiation in this case is usually called *beamstrahlung.* Its properties are similar to those for the ordinary synchrotron radiation in a uniform magnetic field (see, e.g., review [2]).

We call a positron bunch *short* if $\eta \ll 1$. In this case the motion of the electron can be assumed to remain rectilinear over the course of the collision. The radiation in the field of a short bunch is quite different from the synchrotron one. In some respect it is similar to the ordinary bremsstrahlung, which is why we called it *coherent bremsstrahlung.* A classical approach to CBS was given in Ref. [3] (see also the references cited therein). A quantum treatment of CBS and applications of CBS to the modern and future colliders were given recently in Refs. [4,5].

In most colliders either $\eta \ll 1$ (all the pp, $\bar{p}p$, and relativistic heavy-ion colliders, some e^+e^- colliders, and some B factories) or $\eta \sim 1$ (e.g., LEP, Tristan, and some B factories) and only the linear e^+e^- colliders have $\eta \gg 1$. Therefore, the CBS has a very wide region of applicability.

II. DISTINCTIONS OF THE CBS FROM THE USUAL BREMSSTRAHLUNG AND FROM THE BEAMSTRAHLUNG

For the usual bremsstrahlung the number of photons emitted by electrons is proportional to the number of electrons and positrons:

$$dN_\gamma \;\propto\; N_e\, N_p\, dE_\gamma/E_\gamma. \quad (2)$$

In contrast, the number of CBS photons

$$dN_\gamma \;\propto\; N_e\, N_p^2\, dE_\gamma/E_\gamma \quad \text{at} \quad E_\gamma \lesssim E_c = 4\gamma_e^2\hbar c/\sigma_z. \quad (3)$$

Indeed, when a photon energy decreases, the coherence length $\sim 4\gamma_e^2\hbar c/E_\gamma$ becomes comparable to the length of the positron bunch σ_z and at $E_\gamma \lesssim E_c = 4\gamma_e^2\hbar c/\sigma_z$ radiation is caused by the interaction of an electron with the positron bunch as a whole, but not with each positron separately. In these conditions the positron bunch is similar to a "particle " with the charge eN_p. As a result, a probability of radiation is proportional to N_p^2 and the number of photons emitted is proportional to $N_eN_p^2$.

CBS differs from the beamstrahlung first of all due to the soft part of its spectrum. In this region, for CBS (as

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well as for the ordinary bremsstrahlung) $dN_\gamma \propto dE_\gamma/E_\gamma$. Therefore, the total number of CBS photons is infinite, in contrast to beamstrahlung for which (as well as for synchrotron radiation) the total number of photons is finite.

III. EXPERIMENTAL STATUS AND APPLICATIONS

The experimental status of the above-mentioned types of radiation is quite different. The ordinary bremsstrahlung is a well-known process. At the e^+e^- and ep colliders its cross section is large enough and it often is an unwanted background. On the other hand, its large cross section and small angular spread of photons allows one to use this radiation for measuring one of the important parameters of a collider—luminosity (for example, at the HERA and LEP colliders). The beamstrahlung was observed in a single experiment at SLC (Ref. [6]), where it was shown that it can be used for measuring transverse bunch size.

The main characteristic of CBS was calculated only recently; experiments for its observation are planned, but have not yet been performed. Therefore, one can speak about applications of CBS on the preliminary level only. Nevertheless, even now we can see such features of CBS which can be useful for applications. They are the following.

A huge number of the soft photons whose spectrum is determined by the length of the positron bunch are emitted. The number of CBS photons for *a single collision* of the beams is (see the Appendix for details)

$$dN_\gamma = N_0 \Phi(E_\gamma/E_c)\frac{dE_\gamma}{E_\gamma}, \qquad (4)$$

where for the flat Gaussian bunches (e.g., at $a_y^2 = \sigma_{ey}^2 + \sigma_{py}^2 \ll a_x^2 = \sigma_{ex}^2 + \sigma_{px}^2$)

$$N_0 = \frac{8}{3\pi}\,\alpha N_e \left(\frac{r_e N_p}{a_x}\right)^2 \times \frac{\arcsin(\sigma_{ex}/a_x)^2 + \arcsin(\sigma_{ey}/a_y)^2}{[1-(\sigma_{ex}/a_x)^4]^{1/2}} \qquad (5)$$

and the function

$$\Phi(x) = \frac{3}{2}\int_0^\infty \frac{1+z^2}{(1+z)^4}\,\exp[-x^2(1+z)^2]\,dz,$$

$$\Phi(x) = 1 \text{ at } x \ll 1,$$

$$\Phi(x) = (0.75/x^2)e^{-x^2} \text{ at } x \gg 1 \qquad (6)$$

(the values of this function are given in Refs. [4]). For B factories the typical numbers are (see Table I, all numbers for the calculation are taken from Ref. [7])

$$E_c \sim 10 \text{ keV}, \quad N_0 \sim (10^7\text{–}10^8).$$

TABLE I. For the B factories several typical values are listed.

Quantity	PEP-II	Tristan-B	Cesr-B
E_e (GeV)	9(3.1)	8(3.5)	8(3.5)
E_c (keV)	24(3)	40(7)	20(4)
$10^7\ N_0$	12(8)	2(0.8)	25(10)
η	0.9(0.6)	0.6(0.3)	1(0.5)

Specific features of CBS—a sharp dependence of the spectrum (4) on the positron bunch length, an unusual behavior of the CBS photon rate dependent on the impact parameter between axes of the colliding bunches, an azimuthal asymmetry, and polarization of photons—can be very useful for an operative control over collisions and for measuring bunch parameters. [One can determine the positron bunch length by measuring the photon spectrum. The spectrum (6) is valid for the Gaussian beams. In the general case, the photon spectrum is determined by the squared form factor of the longitudinal density of the positron bunch. It depends strongly on the form of this density. In particular, had we used a density of the form $(1/2\sigma_z)\exp(-|z|/\sigma_z)$, instead of the Gaussian one, then Eq. (4) would have been valid, but with another form $\Phi(E_\gamma/E_c)$; in fact, at $E_\gamma/E_c \gg 1$, in this case $\Phi(x) = (8/35)x^{-4}$.]

The last line of Table I gives us values of the parameter η [see Eq. (1)]. It is seen that B factories have either short ($\eta \ll 1$) or intermediate ($\eta \sim 1$) bunches. Strictly speaking, for intermediate bunches our results are only estimates. However, we have reason to believe (see Ref. [4]) that the real parameter for short bunches is determined not by the relation $\eta \ll 1$, but by the relation $\eta \ll 10$, and a short bunch approximation may not be bad for the intermediate case also. Moreover, properties such as a dependence on the impact parameter of the bunches (see Sec. IV) are the same for the short and long bunches.

IV. COLLISIONS WITH THE NONZERO IMPACT PARAMETER OF THE BUNCHES

If the electron bunch axis is shifted in the vertical direction by a distance R_y from the positron bunch axis, the luminosity $L(R_y)$ (as well as the number of events for the usual reactions) decreases very quickly:

$$L(R_y) = L(0)\exp(-R_y^2/4\sigma_y^2). \qquad (7)$$

In contrast, for all the colliders in Table I, the number of CBS photons increases almost two times at $R_y \sim (3-4)\sigma_y$. The corresponding curves for PEP-II and Tristan-B are presented in Fig. 1 [for the calculations we used formulas (A1)–(A5) from the Appendix]. For PEP-II the increase reaches 75% at $R_y = 3\sigma_y$, for Tristan-B, 93% at $R_y = 4\sigma_y$. After that, the rate of photons decreases, but even at $R_y = 15\sigma_y$ the ratio $dN_\gamma(R_y)/dN_\gamma(0) = 1.01$ and 1.66, respectively.

The effect does not depend on the photon energy. It

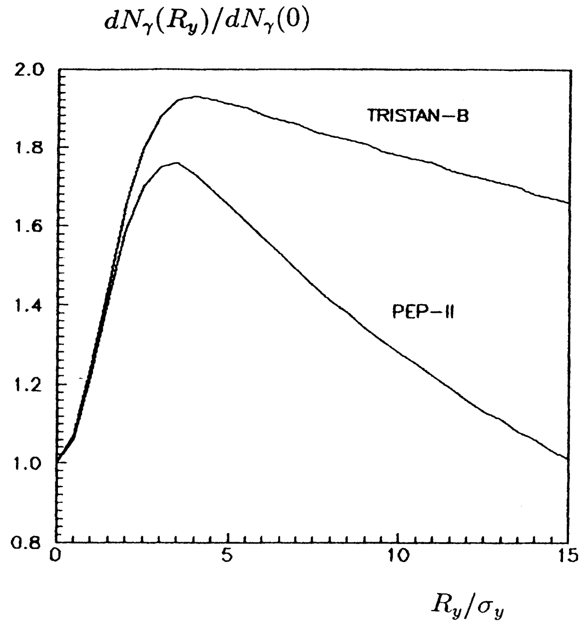

FIG. 1. Ratio of the number of CBS photons $dN_\gamma(R_y)/dE_\gamma$ for a vertical distance between beam axes R_y to that $dN_\gamma(0)/dE_\gamma$ at $R_y = 0$ vs R_y/σ_y, where σ_y is the vertical size of the bunch.

can be explained in the following way.

At $R_y = 0$ a considerable portion of the electrons moves in the region of small impact parameters where electric and magnetic fields of the positron bunch are small. For R_y such as $\sigma_y \ll R_y \ll \sigma_x$, these electrons are shifted into the region where the electromagnetic field of the positron bunch are larger and therefore the number of emitted photons increases. For large R_y (at $\sigma_x \ll R_y \ll \sigma_z$), fields of the positron bunch are $|\mathbf{E}| \approx |\mathbf{B}| \propto 1/R_y$ and therefore $dN_\gamma \propto 1/R_y^2$, i.e., the number of emitted photons decreases, but very slowly.

This feature of CBS can be used for a fast control over impact parameters between beams (especially at the beginning of every run) and over transverse beam sizes. For the case of long bunches, such an experiment has already been performed at SLC (see Ref. [6]).

V. AZIMUTHAL ASYMMETRY AND POLARIZATION

If the impact parameter between beams is nonzero, an azimuthal asymmetry of the CBS photons appears, which can also be used for operative control over beams. For definiteness, let the electron bunch axis be shifted in the vertical direction by the distance R_y from the positron bunch axis. When R_y increases, the electron bunch is shifted into the region where the electric field of the positron bunch is directed almost in a vertical line. As a result, the equivalent photons (produced by the positron bunch) obtain a linear polarization in the vertical direction. The mean degree of such a polarization l for PEP-II and Tristan-B is presented in Fig. 2.

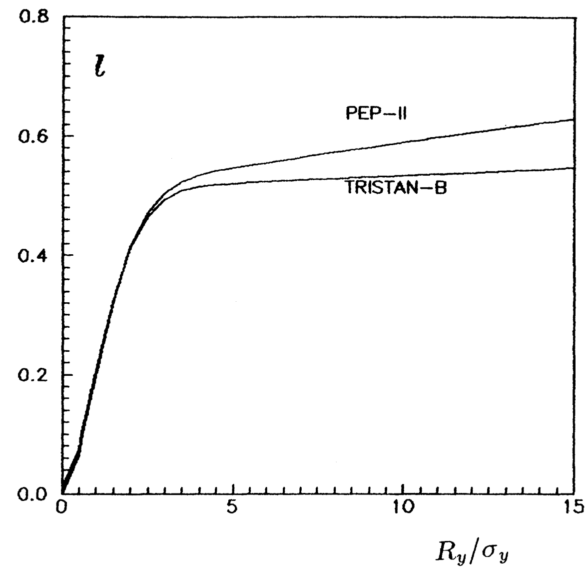

FIG. 2. Degree of the equivalent photon polarization l vs R_y/σ_y, where R_y is a vertical shift of the electron bunch axis.

Let us define the azimuthal asymmetry of the emitted photons by the relation

$$A = \frac{dN_\gamma(\varphi = 0) - dN_\gamma(\varphi = \pi/2)}{dN_\gamma(\varphi = 0) + dN_\gamma(\varphi = \pi/2)}, \tag{8}$$

where the azimuthal angle φ is measured with respect to the horizontal plane. It is not difficult to obtain from Eqs. (A3) and (A6) that this quantity does not depend on photon energy and is equal to

$$A = \frac{2(\gamma\theta)^2}{1 + (\gamma\theta)^4} l, \tag{9}$$

where θ is the polar angle of the emitted photon. From Fig. 2 one can see that when R_y increases, the fraction of photons emitted in the horizontal direction becomes greater than the fraction of photons emitted in the vertical direction.

If the equivalent photons are linearly polarized (and l is its mean degree), then the CBS photons are also linearly polarized in the same direction. Let $l^{(f)}$ be the mean degree of CBS photon polarization. The ratio $l^{(f)}/l$ varies in the interval from 0.5 to 1 when E_γ increases (see Table II).

TABLE II. The ratio $l^{(f)}/l$ for several values of E_γ/E_c.

E_γ/E_c	$l^{(f)}/l$
0	0.5
0.2	0.7
0.4	0.81
0.6	0.86
0.8	0.89
1	0.94
1.5	0.96
2	0.97

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APPENDIX: FORMULAS USED FOR THE CALCULATION OF CBS

We only consider the case when one can neglect the change in the transverse sizes of the bunches during the collision. We use a reference frame in which electrons and positrons have head-on collisions and choose the z axis along the momentum of the initial electron, the x axis in the horizontal direction, and the y axis in the vertical direction. Let θ and φ be the polar and azimuthal angles of the CBS photon.

In Ref. [5] it was shown that with a good enough accuracy CBS can be described in an approximation such that the collective electromagnetic field of the positron bunch may be interpreted as a flux of equivalent photons along the $(-z)$ axis distributed with some density on a frequency spectrum. The spectral expansion of the positron bunch field contains the frequencies $\omega = \mathbf{q}\mathbf{v}_p$ and the spectral components of this field at distance $\mathbf{b}$ from the z axis are

$$\mathbf{E}_\omega(\mathbf{b}) = -\frac{4\pi i e}{v_p}\int \frac{\mathbf{q}_\perp}{\mathbf{q}_\perp^2}\, F_p(\mathbf{q})e^{i\mathbf{q}_\perp\mathbf{b}}\,\frac{d^2q_\perp}{(2\pi)^2},$$

$$\mathbf{B}_\omega(\mathbf{b}) = \frac{\mathbf{v}_p}{c}\times\mathbf{E}_\omega(\mathbf{b}),$$

where $\mathbf{v}_p$ is the positron velocity and

$$F_p(\mathbf{q}) = \int n_p(\mathbf{r})\, e^{-i\mathbf{q}\cdot\mathbf{r}}\, d^3r$$

is the form factor of the positron bunch [or the Fourier transform of the positron bunch density $n_p(\mathbf{r})$]. It is evident that a tensor $E_{\omega i}E^*_{\omega k}$ determines the density of the equivalent photons and their polarization. After averaging over bunches this tensor is proportional to

$$J_{ik} = 4\pi\int \frac{q_i q'_k}{\mathbf{q}_\perp^2\mathbf{q}_\perp'^2}\, F_p(\mathbf{q})F_p^*(\mathbf{q}')F_e(\mathbf{q}'-\mathbf{q})\,\frac{d^2q_\perp d^2q'_\perp}{(2\pi)^4},\quad i,k=x,y, \tag{A1}$$

where F_e is the form factor of the electron bunch and $\omega = \mid q_z \mid c = \mid q'_z \mid c$.

The number of the CBS photons for a single collision of the bunches is equal to

$$dN_\gamma = \frac{\alpha}{\pi}\frac{d\omega}{\omega}J(\omega)d\sigma_C(\omega, E_\gamma),$$

where $d\sigma_C$ is the cross section for Compton scattering of the equivalent photon with the frequency ω on the electron and

$$J(\omega) = J_{xx} + J_{yy}. \tag{A2}$$

The equivalent photon energy $\hbar\omega$ is connected with the energy E_γ and the emission angle θ of the emitted photon by the simple kinematical relation

$$\hbar\omega = [1 + (\gamma_e\theta)^2]\frac{E_\gamma}{4\gamma_e^2}.$$

If the equivalent photons are not polarized and the polarization of the CBS photons are not detected, the Compton cross section is

$$d\sigma_C = 2\pi r_e^2\frac{dE_\gamma}{E_\gamma}\frac{1}{1+z}\frac{d\varphi}{2\pi}\Phi_0,$$

$$\Phi_0 = 2\frac{1+z^2}{(1+z)^2},\qquad z = (\gamma_e\theta)^2.$$

As a result, we obtain the energy and angular distribution of the CBS photons

$$dN_\gamma = \frac{3}{4}N_0\frac{dE_\gamma}{E_\gamma}\frac{dz}{(1+z)^2}\frac{d\varphi}{2\pi}\Phi_0\frac{J(\omega)}{J(0)}, \tag{A3}$$

where the dimensionles constant N_0 is defined as

$$N_0 = \frac{8}{3}\,\alpha\, r_e^2\, J(0). \tag{A4}$$

For the Gaussian beams we have

$$J(\omega)/J(0) \;=\; \exp[-(\omega\sigma_z/c)^2] \tag{A5}$$

from which it is follows that spectrum of the CBS photons has the form of Eqs. (4)–(6).

Polarization states of the equivalent and CBS photons can be described by the Stokes parameters ξ_j and ξ'_j, respectively. Among them ξ_2 and ξ'_2 are the mean photon helicities (they describe the circular polarization). The degrees of the linear polarization l and l' are connected with the Stokes parameters by the relations

$$\xi_1 = -l\sin 2\gamma,\ \xi_3 = l\cos 2\gamma,$$

$$\xi'_1 = l'\sin 2\gamma',\ \xi'_3 = l'\cos 2\gamma',$$

where γ and γ' are the azimuthal angles of the directions of the linear polarizations. [For the equivalent photons we should have taken into account that they move along the $(-z)$ axis and therefore the natural reference frame for them coincides with our one after reflecting the z and y axes.] The polarization vector of the equivalent photon is

$$\mathbf{e} = \frac{\mathbf{E}_\omega}{|\mathbf{E}_\omega|};$$

therefore, the averaged density matrix of the equivalent photons is

$$\langle e_i e_k^* \rangle = \frac{J_{ik}}{J(\omega)} = \frac{1}{2}\,(1 - \xi_1\sigma_1 - \xi_2\sigma_2 + \xi_3\sigma_3)_{ik},$$

where σ_n are Pauli matrices.

The Compton cross section for the polarized photons can be obtained from Ref. [8]. In this case the quantity Φ_0 in Eq. (A3) should be replaced by

$$\Phi_0 \to \frac{1}{2}\left(\Phi_0 + \sum_{j=1}^{3}\Phi_j\xi_j'\right),$$

$$\Phi_0 = 2\frac{1+z^2}{(1+z)^2} - \frac{4z}{(1+z)^2}l\cos(2\varphi - 2\gamma),$$

$$\Phi_2 = -2\frac{1-z^2}{(1+z)^2}\xi_2,$$

$$\Phi_1 = \frac{2}{(1+z)^2}[l\sin 2\gamma + z^2 l\sin(4\varphi - 2\gamma) - 2z\sin 2\varphi] \tag{A6}$$

$$\Phi_3 = \frac{2}{(1+z)^2}[l\cos 2\gamma + z^2 l\cos(4\varphi - 2\gamma) - 2z\cos 2\varphi].$$

The Stokes parameters ξ_j' characterize the detected polarization. Let $\xi_j^{(f)}$ be the Stokes parameters for the CBS photons themselves. From Eq. (A6) one can obtain that

$$\xi_j^{(f)} = \frac{\Phi_j}{\Phi_0}.$$

Even if the equivalent photons are not polarized, the CBS photons get a linear polarization in the direction which is perpendicular to the scattering plane:

$$l^{(f)} = \frac{2z}{1+z^2},\quad \gamma^{(f)} = \varphi + \frac{\pi}{2},\quad \xi_2^{(f)} = 0 \quad \text{at} \quad \xi_j = 0.$$

Integrating the distribution (A3) and (A6) over the angles θ and φ, one obtains the energy distribution of CBS photons

$$dN_\gamma = \frac{1}{2}N_0[\Phi(x) + \Phi^l(x)ll'\cos(2\gamma - 2\gamma') + \Phi^c(x)\xi_2\xi_2']\frac{dE_\gamma}{E_\gamma},\quad x = \frac{E_\gamma}{E_c}, \tag{A7}$$

where for the Gaussian beams

$$\Phi(x) = \int_0^\infty (1+z^2)F(x,z)\,dz,$$

$$\Phi^c(x) = -\int_0^\infty (1-z^2)F(x,z)\,dz = \Phi(x) - 2\Phi^l(x),$$

$$\Phi^l(x) = \int_0^\infty F(x,z)\,dz,$$

$$F(x,z) = \frac{3}{2}\frac{\exp[-x^2(1+z)^2]}{(1+z)^4}. \tag{A8}$$

At small photon energy one has

$$\Phi(0) = 1,\quad \Phi^l(0) = \frac{1}{2},\quad \Phi^c(0) = 0$$

and at large energy (at $x \gg 1$) one has

$$\Phi(x) = \frac{3}{4x^2}\left(1 - \frac{5}{2x^2} + \frac{37}{4x^4} + \cdots\right)e^{-x^2};$$

$$\Phi^l(x) = \frac{3}{4x^2}\left(1 - \frac{5}{2x^2} + \frac{35}{4x^4} + \cdots\right)e^{-x^2}.$$

Averaged over θ and φ, the Stokes parameters of CBS photons are

$$\langle l^{(f)}\rangle = \frac{\Phi^l(x)}{\Phi(x)}l,\quad \langle\gamma^{(f)}\rangle = \gamma,\quad \langle\xi_2^{(f)}\rangle = \frac{\Phi^c(x)}{\Phi(x)}\xi_2,\quad x = \frac{E_\gamma}{E_c}. \tag{A9}$$

For all known examples the circular polarization of the equivalent photons is absent, $\xi_2 = 0$; hence, for these examples $\langle\xi_2^{(f)}\rangle = 0$ also.

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